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## Letter

# Higher order modal dynamics of the flexural ultrasonic transducer

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## Abstract

The flexural ultrasonic transducer (FUT) consists of a piezoelectric ceramic bonded to an edge-clamped elastic plate, for both generation and detection of ultrasound waves. It is typically employed for proximity measurement, such as in automotive parking systems, and for flow measurement in gases and liquids. Conventional industrial applications have generally incorporated FUTs with resonance frequencies up to around 50 kHz. However, there have been recent advances in the understanding of the FUT, both in terms of fabrication and operation, enabling the potential for measurement in a wider range of applications, including those of elevated pressure, temperature, and requiring multiple operating frequencies. Ultrasound measurement with FUTs at frequencies greater than 50 kHz is desirable in a range of applications, including gas and water metering in petrochemical plants, district heating, and power industries. The major restricting limitation of designing transducers to operate at these higher frequencies has been a relatively poor understanding of these transducers work, including optimisation of design and performance, and the few reports into how different modes of a FUT can be utilised for practical and reliable measurement. In this study, the higher order modal dynamics of the FUT are investigated through measurement of high frequency ultrasound waves in air, for different fundamental operating modes. A combination of experimental techniques is applied, comprising electrical impedance analysis and laser Doppler vibrometry. The experimental research is supported by analytical solutions to reveal complex higher order modal dynamics of the FUT. This investigation represents further development in widening the industrial application potential of the FUT.

**Keywords:** flexural ultrasonic transducer, higher order modes, high frequency, air-coupled ultrasound

(Some figures may appear in colour only in the online journal)

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The flexural ultrasonic transducer (FUT) has recently been investigated as an *in-situ* measurement device for applications such as flow measurement [1]. It is a robust device consisting of a piezoelectric ceramic attached to a vibrating plate [2]. The device is relatively straightforward to manufacture, requiring few components, and they can be readily tailored for operation in different fluids across gases and liquids. It is a very efficient transducer, normally requiring only a few volts from the drive electronics to generate ultrasound waves which can be detected by another transducer over a propagation distance in the order of 1 m. However, one of the principal limitations of FUT-based measurement technology at present, is that commercially available devices are generally only designed for operation up to around 50 kHz [3], typically at the fundamental axisymmetric (0,0) and (1,0) modes. FUTs are particularly advantageous below 50 kHz in terms of the ultrasound propagation distance over which they can be used for relatively low excitation voltages: in the region of several metres for <10 V. FUTs also couple efficiently to a wide range of fluids without the requirement for matching layers. Their operating frequency is predominantly governed by the physical specifications of the elastic plate, which must be suitably compliant to generate the required amplitude of vibration at the intended operating frequency. There are important reasons that the measurement of ultrasound above a frequency of 50 kHz is of interest, some of which have been highlighted by the authors in prior research into this area [3]. For example, one key advantage of high frequency ultrasound measurement is the improvement in signal resolution. Another benefit is that it presents a reliable mechanism for reducing interference of ultrasound waves which can arise from low frequency noise generated by peripheral structures or components within a measurement system, for example inside a fluid pipeline. High frequency ultrasound measurement in air has been reported [4], but this was limited to relatively large power transducers. The research presented here does not exclusively focus on operating at a higher frequency, the advantages of which are highlighted in prior research [3], but also in a higher order mode. The importance of this research to the ultrasonic measurement community is centred on the understanding of the FUT for reliable acquisition of multiple measurements at different frequencies in a single system. Furthermore, to enable the delivery of improvements to ultrasonic measurement technology for gas and water metering in the energy and petrochemical industries, new strategies for the generation and detection of high frequency ultrasound signals are vital. This is particularly important due to the increase in demand for ultrasound devices capable of operating in high pressure and temperature environments, where the FUT is now exhibiting considerable potential. In this paper, the higher order modal dynamics of the FUT are explored using a combination of experimental and analytical methods. The principal contribution of this study is the demonstration of higher order modal dynamics of the FUT which can be encountered in operation, thereby enabling the future design of optimised configurations which can exploit such modes of vibration. The specific original contributions to knowledge of this research

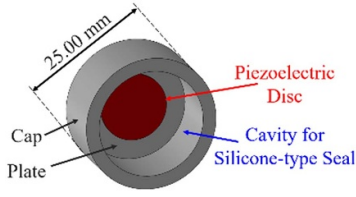
are measurements of coupled higher order modes in FUTs; the explanation of inconsistencies observed between analytical and experimental outputs; and an analytical representation of higher order modal dynamics supporting the experimental observations.

The context for understanding higher order dynamics of the FUT can best be communicated by demonstrating the operating mechanism of the device. The assembly of a typical FUT, such as that used to obtain the experimental results shown in this paper, is shown in figure 1.

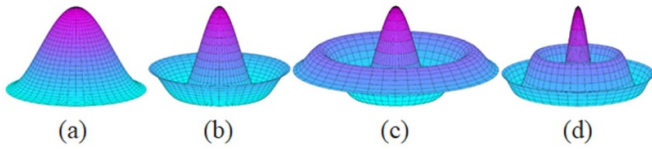
The typical commercial FUT for industrial application is composed of a metal cap, inside which a piezoelectric ceramic disc is attached. Electrode connections are then made on the piezoelectric ceramic disc, before the FUT is sealed at the rear, often with a compliant material such as silicone rubber. It is common for the piezoelectric ceramic disc to be attached to the plate using a high strength epoxy resin and positioned at the centre of the plate, ensuring circular symmetry. This is vital to ensure the FUT exhibits the mode shapes which can be analytically determined, and which are explored in more detail in this paper. It should be noted that electrodynamic FUTs have also been investigated in prior research [5], which eliminate the need for a piezoelectric ceramic disc.

A key limitation associated with piezoelectric ceramics for air-coupled ultrasound measurement is that their acoustic impedances can be in the order of 35 MRayl, significantly higher than that of air, which is approximately 400 Rayl [6]. One strategy to address this limitation is to use matching layers between the piezoelectric and fluid. However, the FUT is a practical solution for ultrasound measurement in different fluids, both gases and liquids, overcoming this acoustic impedance mismatch and which is also capable of the generation and detection of ultrasound at high frequencies. The reason for this is that the device is not subject to the acoustic impedance mismatch limitation, since its flexible plate couples efficiently with the fluid in which it operates, vibrating according to plate modes as demonstrated by Leissa in 1969 [7]. The vibrations of the piezoelectric ceramic disc stimulate these modes in the plate, thereby generating mode shapes which can be mathematically predicted. This is the operating principle of the FUT, and one of the reasons the device is receiving growing interest from the industrial ultrasonic measurement community.

The plate of a FUT can be approximated as having an edge-clamped boundary condition. This is a valid assumption because the bulk mass of the piezoelectric ceramic disc and the epoxy resin used to fabricate the FUT is small compared to that of the metallic plate. Secondly, the side-wall of the FUT cap acts as a circumferential boundary condition on the plate. Using these boundary conditions, the real operating modes of the FUT are observed to be in close correlation with the analytical solutions of the plate modes of vibration, determined using Leissa's theory [7]. The first four axisymmetric modes are illustrated in figure 2 from mathematical simulation, designated as the (0,0), (1,0), (2,0), and (3,0) modes of vibration. The mode nomenclature is aligned with that used in prior research for consistency [8], where an increase in the mode order correlates with a higher resonance frequency.



**Figure 1.** The basic assembly of a FUT.



**Figure 2.** Mathematically simulated axisymmetric mode shapes of a FUT plate from Leissa's theory of plate vibrations [7], showing (a) the (0,0), (b) the (1,0), (c) the (2,0), and (d) the (3,0) modes. Note the modes are not shown with relative amplitude scaling as would be observed in the operation of a FUT optimised for a specific mode.

These mode shapes are an effective and accurate representation of the operating modes of the FUT plate. The FUT is commonly designed to operate around 40 kHz in an axisymmetric mode for commercial applications, such as automotive parking sensors and metrology [9, 10]. These axisymmetric modes are usually either the fundamental or first order as shown by figures 2(a) and (b), which can be closely correlated with the solutions developed by Leissa [7].

In general, the differential equation associated with transverse plate displacement  $w$  can be given by (1), where  $D$  is the rigidity factor, accounting for the thickness of the plate ( $h$ ) and its material properties of Young's modulus ( $E$ ), density ( $\rho$ ) and Poisson's ratio ( $\nu$ ), and the position vector is  $x$ . For reference,  $D$  is shown by equation (2).

$$D\nabla^4 w(x,t) + \rho \frac{\delta^2 w(x,t)}{\delta t^2} = 0 \quad (1)$$

$$D = \frac{Eh^3}{12(1-\nu^2)} \quad (2)$$

After application of boundary conditions for uniform edge-clamping [7, 8], the mode shapes can be simulated, whose associated frequencies can be determined using (3), where  $a$  is the plate radius, and  $m$  and  $n$  denote the nodes used to generate the mode order.

$$f_{m,n} = \frac{1}{2\pi} \left( \frac{\lambda_{m,n}}{a} \right)^2 \sqrt{\frac{D}{\rho}} \quad (3)$$

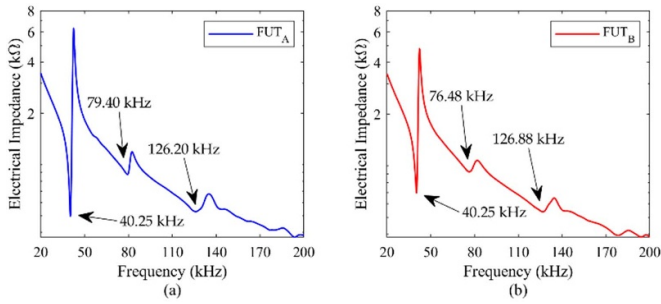
In (3),  $\lambda_{m,n}$  is a mode constant which depends on the radius, flexural rigidity, density, and frequency of the plate, where its derivation can be found in full in Leissa's work [7]. The mode shapes can be configured to align with a 40 kHz resonance

frequency via adjustment of plate diameter and thickness, depending on the elastic properties of the cap material. FUTs which are tailored to operate efficiently around 40 kHz in the (0,0) and (1,0) modes are common, but the higher order (2,0) and (3,0) modes are not exploited. Furthermore, operation at frequencies exceeding 40 kHz has also not been practically demonstrated, except in prior research reporting the use of alternative FUTs [3, 11]. The challenge for utilising the higher order modes of a FUT in a practical application, is demonstrating a strategy for the design of high frequency FUTs. The dynamic characteristics of a FUT have been reported in prior research [2], and there are critical operating parameters to be considered which affect the dynamics of the FUT. For example, the boundary condition applied to the FUT during operation significantly affects both its output amplitude and its resonance frequency, and the material properties and geometrical dimensions of the plate are critical to the resonance frequency and amplitude of the plate.

In this study, a commercial FUT is utilised, predominantly because it is an industrially relevant device. The FUT (Pro-Wave Electronics Corporation) comprises an aluminium cap with a plate diameter of 25.00 mm and nominal thickness of 0.60 mm, with a resonance frequency of  $40 \pm 1.0$  kHz for the fundamental operating mode. The PZT disc which is attached to the plate and is housed inside the aluminium casing has a nominal diameter of 9.00 mm, and a thickness of approximately 0.30 mm. The FUT incorporates a silicone rubber-type backing, with an air gap immediately behind the PZT disc. Although this investigation examines the dynamics of this FUT only, it is intended to inform the design and operation of FUTs, tailored to different specifications. Electrical impedance analysis (EIA, Agilent 4294 A impedance gain/phase analyzer) is used in conjunction with the fundamental analytical theory to determine the resonance frequencies of the operating modes. The axisymmetric operating modes of interest are then measured accounting for this information using laser Doppler vibrometry (LDV, Polytec OFV-5000, with the OFV-505 sensor head). The probe laser spot size has a diameter of approximately  $10 \mu\text{m}$  at the stand-off distance used and the controller provides a maximum sensitivity of approximately  $3 \mu\text{m s}^{-1}$ . The axisymmetric resonance frequency for each mode measured through LDV is identified by focusing the laser at the centre of the subject FUT's plate, where the drive frequency is modulated around the expected frequency of that mode, using EIA results as a guide. Once the modal frequency is identified, a measurement window is established, which encompasses the entirety of the FUT plate. MATLAB is used to control the positioning of the LDV measurement laser, where amplitude-time blocks are recorded in sequence, and in steps of 1 mm, until the entire plate surface is scanned. These blocks are then combined in MATLAB to generate the mode shape motion.

The physical characteristics of the plate, as referenced above, can be applied in the system of equations proposed by Leissa to provide an estimate of the resonant frequencies of the (0,0), (1,0), (2,0), and (3,0) modes, by assuming the plate is an edge-clamped plate [7, 8]. Using (1)–(3) and the physical specifications of the FUTs, these modal frequencies



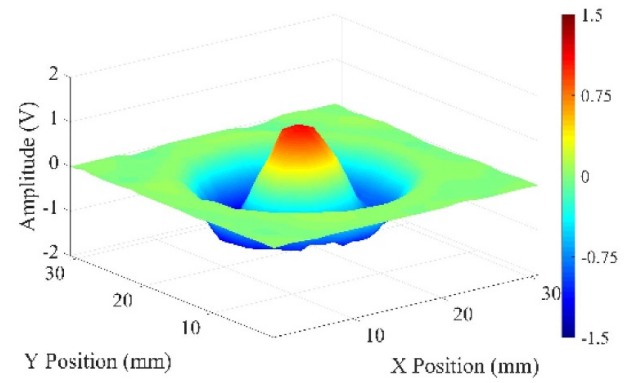


**Figure 3.** EIA spectra showing the resonance frequencies of (a)  $FUT_A$  and (b)  $FUT_B$ , extracted from the series resonance frequencies at the corresponding local minimum of electrical impedance.

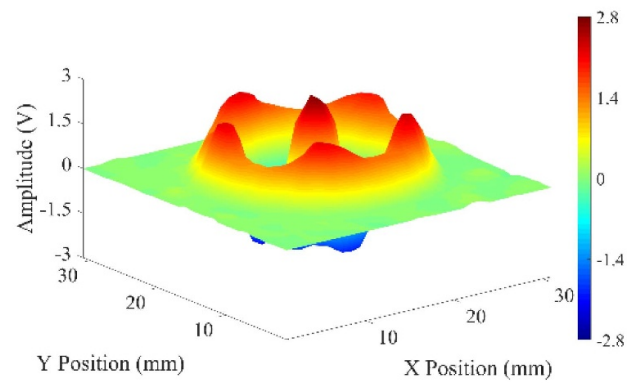
were calculated to be 9.65 kHz, 37.58 kHz, 84.19 kHz, and 149.45 kHz respectively. It should be noted that the modal frequencies will not precisely match those observed in practice, because they are derived from analytical solutions based on an edge-clamped plate and are not a multi-physics simulation of a transducer. The application of Leissa's theory to determine mode shapes and resonance frequencies, aligned to finite element calculations, has already been demonstrated for the FUT in prior research [11], and is the justification for utilising Leissa theory here to explain the nature of the measured modes. The identification of the associated mode shapes is therefore possible when using this technique. The advantage of optical measurement such as LDV to characterise FUTs in higher order modes is also demonstrated. The spectra for two nominally similar FUTs,  $FUT_A$  and  $FUT_B$ , from EIA are shown in figure 3, and mode shapes corresponding with those resonance frequencies shown in figure 4 for  $FUT_A$ , using a function generator drive voltage of 10 V<sub>P-P</sub>.

Here, EIA measures the electromechanical response of the FUT, whereas LDV detects plate motion. There can hence be differences in measured resonance frequencies, but LDV is particularly useful at obtaining mode shapes at high measurement resolution. In particular, the measurement sensitivity of LDV allows high resolution capture of mode shapes, for which the FUT is not optimised, for example exhibiting lower amplitudes. The drive conditions associated with EIA are also different to LDV, where the former drive voltage can be significantly lower, around 0.50 V<sub>RMS</sub>. Hence there can be influences from dynamic nonlinearity for methods requiring elevated drive voltages [12]. Nevertheless, the (1,0) mode, observed in figure 4(a), generally conforms to the modal characteristics mathematically generated in figure 2(b). It should also be noted that differences between the resonance frequencies of nominally similar FUTs can occur, because the dynamics of the FUT are sensitive to the applied boundary condition, and sub-millimetre variations in geometry, principally plate diameter and thickness.

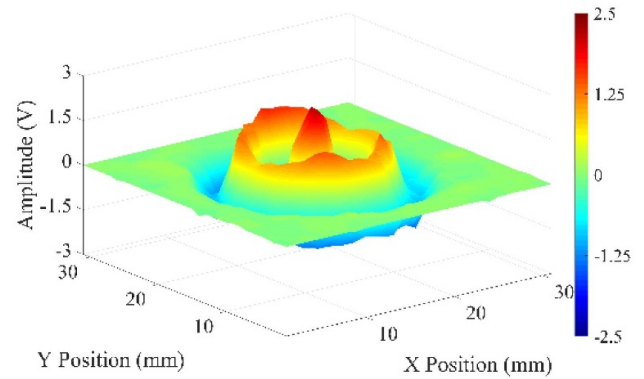
The key observation of interest is that the mode shapes displayed in figures 4(b) and (c) do not precisely align with those shown in figures 2(c) and (d). There is strong evidence of influence from other modes in each result, via the non-uniform ring shapes in each mode. The hypothesis was made here



(a)



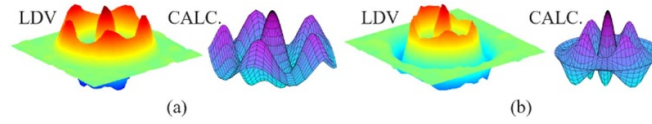
(b)



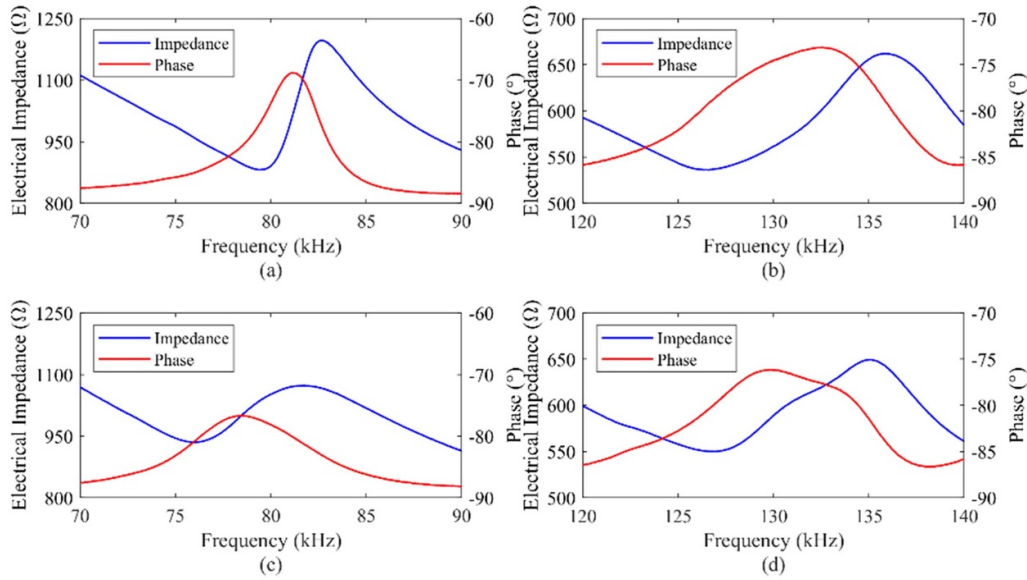
(c)

**Figure 4.** LDV mode shapes of  $FUT_A$ 's plate, for (a) the (1,0) at 40.25 kHz for a measurement sensitivity of 100 mm s<sup>-1</sup> V<sup>-1</sup>, (b) a (2,0)-like at 81.00 kHz for a sensitivity of 10 mm s<sup>-1</sup> V<sup>-1</sup>, and (c) a (3,0)-like at 139.30 kHz for a sensitivity of 10 mm s<sup>-1</sup> V<sup>-1</sup>. The amplitudes shown in the colour bar are given in terms of voltage and are directly related to the velocity measurements as referenced by the stated measurement sensitivities.

that an asymmetric (0,5) mode may exist close to the (2,0) mode, with a (1,4) mode existing close to the (3,0) mode, based on the mode shape characteristics and the modal frequency estimations. A solution to (1),  $W_S$ , was then computed of the form shown by (4), to determine if these modes could be identified through the analytical relationships, accounting for modal superposition.



**Figure 5.** The mode shapes for (a) around the (2,0) mode, and (b) around the (3,0) mode, via LDV and analytical superposition using (3).



**Figure 6.** EIA spectra around the frequency ranges of interest for those modes shown in figure 4, for (a)  $FUT_A$  around the (2,0) mode, (b)  $FUT_A$  around the (3,0) mode, (c)  $FUT_B$  around the (2,0) mode, and (d)  $FUT_B$  around the (3,0) mode.

$$W_S(r, \theta) = \left( A_n I_n \left( \frac{\lambda_{M1}}{a} r \right) + B_n J_n \left( \frac{\lambda_{M1}}{a} r \right) \right) \cos(n\theta) + \left( A_n I_n \left( \frac{\lambda_{M2}}{a} r \right) + B_n J_n \left( \frac{\lambda_{M2}}{a} r \right) \right) \quad (4)$$

where,  $A$  and  $B$  are constants from the edge-clamped boundary condition on the plate,  $I$  and  $J$  are Bessel functions,  $\lambda$  is the mode constant referred to previously, which is dependent on the mode order, and  $M1$  and  $M2$  are the asymmetric and axisymmetric modes of interest, respectively. The constituent mode shapes of the coupled mode were first calculated for the physical parameters of the plate before these modes were mathematically superpositioned. For example, the (0,5) and (2,0) modes were individually calculated, before the combined mode shape was generated. The (0,5) mode frequency was calculated to be 85.73 kHz, only 1.54 kHz higher than the predicted frequency of the (2,0) mode. The (1,4) mode was calculated to be 132.37 kHz, which is 17.08 kHz lower than the predicted frequency of the (3,0) mode. This shows that modal coupling is likely to occur, where operation of the FUT in the nominal axisymmetric modes stimulates the proximate asymmetric modes. The coupled (0,5)-(2,0) and (1,4)-(3,0) mode shapes were then analytically simulated according to (3) to allow superposition, with both shown in figure 5 alongside the real mode shapes captured optically using LDV.

A close correlation exists between analytical relationships attributed to plate modes which are used to represent the FUT, with observations by experiment. The coupling between the (0,5) and (2,0) appears strong, which is to be expected, given the calculated proximity in frequency of the two resonant modes. In contrast, there appears to be only weak coupling between the (1,4) and (3,0) modes, but again this is expected given the larger separation in frequency between these resonant modes, compared to the much closer (0,5)-(2,0) mode combination. To investigate if these modes could be identified independently in the frequency domain, the examination of the impedance-frequency responses for the modes shown in figure 4 was then conducted, using EIA at higher resolution than the results shown in figure 3. This was undertaken to investigate the presence of coupling in the vicinity of the (2,0) or (3,0) modes and assess if it is possible to distinguish the modal coupling behaviour which would align with the spatial domain results shown in figures 4 and 5. The EIA spectra are shown in figure 6.

For both transducers, only one resonant peak is identifiable close to the (2,0) and (3,0) modes as presented in figures 4 and 5. This proximity can be quantified in terms of the estimated frequency separation as calculated through the analytical relationships, for example 1.54 kHz in the case of the (0,5) and (2,0) modes. There are evidently single-mode responses

around these axisymmetric modes, as is evident when considering the results obtained using EIA, and this does not align with the responses measured in the spatial domain through LDV as presented in figures 4 and 5. LDV scans in the spatial domain are therefore a more practical and reliable method for visualizing and quantifying modal performance for operating FUTs in higher order modes.

In general, the analytical calculations correlate well with experimental observations using LDV, which is useful for future application of the higher order modes present in FUTs. There are however significant challenges which must be overcome. The first is that two FUTs that are used for pitch-catch ultrasound measurement should be suitably matched in their responses. The ultrasound waves generated from the modal behaviour of one FUT in transmission will convolve with the dynamics of a second FUT used as the detector. This will be complicated by modal coupling in the spatial domain in operation, and must be considered for future optimisation. However, the central impact of this research is that higher order, non-optimised modes of a FUT can be understood and utilized through modifications to Leissa's analytical relationships governing the dynamics of edge-clamped plates, considered together with LDV measurements. This research has shown that non-optimised higher order modes can display interactions and coupling not previously identified, therefore is significant for how FUTs can be optimised in future as multi-modal devices. The limitations of analysis in frequency space compared to the spatial domain have also been demonstrated, using EIA in combination with LDV. This is transformative for realising FUTs as readily tunable multi-mode ultrasound measurement devices. The FUT thus exhibits significant potential for a wide range of industrial measurement applications, and versatile, multi-frequency, *in-situ* ultrasound measurement.

The wider implications of this research include the opportunity to configure complex mode shapes for multi-mode ultrasound measurement, and the ability to generate several functional operating modes in a single FUT by modulating the drive parameters. It would also be possible to control the dynamic performance of both transmission and detection FUTs together, and this will be investigated in detail as part of future research.

### Data availability statement

The data that support the findings of this study are openly available at the following URL/DOI: [warwick.ac.uk/fac/sci/physics/research/ultra/research/hiffut/](http://warwick.ac.uk/fac/sci/physics/research/ultra/research/hiffut/).

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